

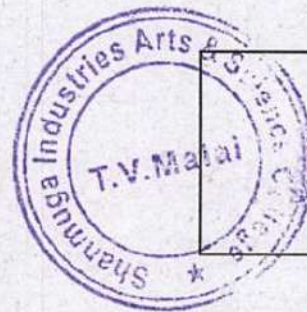
SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Lebesgue dominated convergence theorem.
17. Let $\{g_n\}$ be a sequence of functions in $L^2(I)$ such that the series $\sum_{n=1}^{\infty} \|g_n\|$ converges. Prove that the series of function $\sum_{n=1}^{\infty} g_n$ converges almost everywhere on I to a function g in $L^2(I)$ and

$$\|g\| = \lim_{n \rightarrow \infty} \left\| \sum_{k=1}^n g_k \right\| \leq \sum_{k=1}^{\infty} \|g_k\|.$$

18. State and prove Jordan theorem.
19. If both partial derivatives $D_r f$ and $D_k f$ exist in an n -ball $B(c; \delta)$ and if both are differentiable at c , prove that $D_{r,k} f(c) = D_{k,r} f(c)$.
20. State and prove the second derivative test for extrema.



APRIL/MAY 2025

GMA22 — REAL ANALYSIS – II

Time : Three hours

Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define upper function.
2. Write down any two properties of Lebesgue integral.
3. Give an example for improper Riemann integral.
4. Define L^2 -norm of f .
5. State Bessel's inequality.
6. Write down the statement of Dini's theorem.
7. What is called first order Taylor formula?
8. Write the chain rule.
9. State inverse function theorem.
10. Define saddle point.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) If f and g are in $L(I)$, prove that the functions f^+ , f^- , $|f|$, $\max(f, g)$ and $\min(f, g)$ are in $L(I)$. Also prove that

$$\left| \int_I f \right| \leq \int_I |f|.$$

Or

- (b) Let I be an interval which is the union of two subintervals, say $I = I_1 \cup I_2$, where I_1 and I_2 have no interior points in common. If $f \in L(I)$. Prove that $f \in L(I)$, $f \in L(I_2)$ and $\int_I f = \int_{I_1} f + \int_{I_2} f$.

12. (a) Let f be defined on the half-infinite interval $I = [a, +\infty)$. Assume that f is Lebesgue-integrable on the compact interval $[a, b]$ for each $b \geq a$, and that there is a

positive constant M such that $\int_a^b |f| \leq M$ for all $b \geq a$. Prove that $f \in L(I)$, the limit

$$\lim_{b \rightarrow \infty} \int_a^b f \text{ exists and } \int_a^{+\infty} f = \lim_{b \rightarrow +\infty} \int_a^b f.$$

Or

- (b) If f, g are in $L^2(I)$ prove that

(i) $|(f, g)| \leq \|f\| \|g\|$ (Cauchy-Schwarz inequality)

(ii) $\|f + g\| \leq \|f\| + \|g\|$ (triangle inequality)

13. (a) State and prove Parseval's formula.

Or

- (b) State and prove Dini's theorem.

14. (a) If f is differentiable at c , prove that f is continuous at c .

Or

- (b) Let S be an open connected subset of R^n , and let $f: S \rightarrow R^m$ be differentiable at each point of S . If $f'(C) = 0$ for each c in S , prove that f is constant on S .

15. (a) Let A be an open subset of R^n and assume that $f: A \rightarrow R^n$ has continuous partial derivatives $D_i f_i$ on A . If $J_f(x) \neq 0$ for all x in A , prove that f is an open mapping.

Or

- (b) A quadric surface with center at the origin has the equation

$$Ax^2 + By^2 + Cz^2 + 2Dyz + 2Ezx + 2Fxy = 1.$$

Find the lengths of its semi-axes.

